



HANDOUT FOR OPEN BOOK EXAMINATION
FOR C-21 III SEMESTER
SC-301-APPLIED ENGINEERING MATHEMATICS
UNIT-I

Integration.

Integration is the inverse process of differentiation.

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
2. $\int \frac{1}{x} dx = \log x + C$
3. $\int a^x dx = \frac{a^x}{\log a} + C$
4. $\int e^x dx = e^x + C$
5. $\int \sin x dx = -\cos x + C$
6. $\int \cos x dx = \sin x + C$
7. $\int \tan x dx = \log |\sec x| + C$
8. $\int \cot x dx = \log |\sin x| + C$
9. $\int \sec x dx = \log |\sec x + \tan x| + C$ or $\log \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right| + C$
10. $\int \operatorname{cosec} x dx = \log |\operatorname{cosec} x - \cot x| + C$ or $\log \left| \tan \left(\frac{x}{2} \right) \right| + C$
11. $\int \sec^2 x dx = \tan x + C$
12. $\int \operatorname{cosec}^2 x dx = -\cot x + C$
13. $\int \sec x \tan x dx = \sec x + C$
14. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + C$
15. $\int \sinh x dx = \cosh x + C$
16. $\int \cosh x dx = \sinh x + C$
17. $\int \operatorname{sech}^2 x dx = \tanh x + C$
18. $\int \operatorname{cosech}^2 x dx = -\coth x + C$
19. $\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$
20. $\int \operatorname{cosech} x \coth x dx = -\operatorname{cosech} x + C$



21. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
22. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$
23. $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + C$
24. $\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1} x + C$
25. $\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C$

Formulae for integration by substitution

1. $\int f^1(ax+b)dx = \frac{f(ax+b)}{a} + C$
2. $\int f(g(x))g'(x)dx$ [Hint put $g(x)=t$]
3. $\int f(x^n)x^{n-1}dx$ [Hint put $x^n = t$]
4. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + C$
5. $\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + C$
6. $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C$
7. $\int \sin^m x \cos^n x dx$ where m and n are positive integers
 - Case (i): when 'm' is odd then put $\cos x=t$
 - Case (ii): when 'n' is odd then put $\sin x=t$
 - Case (iii): when both 'm' and 'n' are odd and $m < n$ then put $\cos x=t$ and $m > n$ then put $\sin x=t$.
 - Case (iv): when m and n are small positive even integers then use $\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$



Nine Standard integrals:

1. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$
2. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$
3. $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$
4. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + c$
5. $\int \frac{1}{\sqrt{x^2-a^2}} dx = \cosh^{-1} \frac{x}{a} + c \text{ (or) } \log |x + \sqrt{x^2 - a^2}| + c$
6. $\int \frac{1}{\sqrt{a^2+x^2}} dx = \sinh^{-1} \frac{x}{a} + c \text{ (or) } \log |x + \sqrt{a^2 + x^2}| + c$
7. $\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$
8. $\int \sqrt{a^2+x^2} dx = \frac{x}{2} \sqrt{a^2+x^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + c$
9. $\int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a} + c$

Integrals of the form

$$\int \frac{1}{ax^2+bx+c} dx, \int \frac{1}{\sqrt{ax^2+bx+c}} dx \text{ and } \int \sqrt{ax^2+bx+c} dx$$

Hint : Express $ax^2 + bx + c$ as sum or difference of two squares then use above nine standard integrals.

Integrals of the form $\int \frac{1}{a \pm b \sin x} dx, \int \frac{1}{a \pm b \cos x} dx, \int \frac{1}{a \cos x \pm b \sin x \pm c} dx :$

Put $\tan \frac{x}{2} = t$, so that

$$dx = \frac{2}{1+t^2} dt, \sin x = \frac{2t}{1+t^2} \text{ and } \cos x = \frac{1-t^2}{1+t^2}$$



UNIT-II

Indefinite Integration-II

1. Integration by using partial fractions.

To resolve given proper rational fraction into partial fractions using the following table

S. No.	Form of the proper fraction	Form of the partial fraction
1	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
2	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
3	$\frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
4	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
5	$\frac{px^2+qx+r}{(x-a)(ax^2+bx+c)}$	$\frac{A}{(x-a)} + \frac{Bx+C}{(ax^2+bx+c)}$ Where A,B and C are constants to be determined and ax^2+bx+c is an irreducible quadratic polynomial.

Note: If the given fraction is improper, then divide the numerator with denominator and use the following:

$$\frac{\text{Numerator}}{\text{Denominator}} = \text{quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

INTEGRATION BY PARTS

$$\mathbf{1.} \int UV \, dx = U \int V \, dx - \int \left(\frac{dU}{dx} \int V \, dx \right) dx$$

To choose U we follow the Order '**ILATE**'

ILATE means

I-Inverse trigonometric function (Ex: $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ etc)

L-Logarithmic functions (Ex: $\log x$, $\log x^2$ etc)

A-Algebraic functions (Ex: x^0 , x^2 , $(x-x^2)$ etc)



T-Trigonometric function (Ex: $\sin x$, $\cos x$, $\tan \theta$, $\sec^2 a$ etc)

E-Exponential function (Ex: e^x , a^x , e^{2x} etc)

2. Bernoulli's Rule

$$\int UV dx = UV_1 - U'V_2 + U''V_3 - U'''V_4 + \dots$$

1. Where $U, U', U'', \dots$ are successive derivatives of U ,

2. $V_1, V_2, V_3, \dots$ are successive integrals of V

$$3. \int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c$$

$$4. \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c$$

$$5. \int e^x (f(x) + f'(x)) \, dx = e^x f(x) + c$$

UNIT-III

Definition: $\int_a^b f(x) dx = [g(x)]_a^b = g(b) - g(a)$

where $\frac{d}{dx} [g(x)] = f(x)$

Properties:

$$1. \int_a^b f(x) dx = \int_a^b f(t) dt$$

$$2. \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$3. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \quad a < c < b.$$

$$4. \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$5. \int_0^a f(x) dx = \frac{1}{2} \int_0^a [f(x) + f(a-x)] dx$$

$$6. \int_{-a}^a f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(x) \text{ is an even function} \\ 0, & \text{if } f(x) \text{ is odd function} \end{cases}$$

$$7. \int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$$



UNIT -IV

1. The area bounded by the curve $y = f(x)$, the x-axis and the lines at $x=a$ and $x=b$ is given by $\left| \int_a^b y \, dx \right|$
2. The area bounded by the curve $x = g(y)$, the y-axis and the lines at $y=c$, and $y=d$ is given by $\left| \int_c^d x \, dy \right|$
3. The area bounded by the curves $y_1 = f(x)$ and $y_2 = g(x)$ and two lines $x=a$ and $x=b$ is given by $\left| \int_a^b (y_1 - y_2) \, dx \right|$
4. The area bounded by the curves $x_1 = f(y)$ and $x_2 = g(y)$ and two lines $y=c$ and $y=d$ is given by $\left| \int_c^d (x_1 - x_2) \, dy \right|$

VOLUME OF SOLID OF REVOLUTION

- (a) The volume of the solid generated by the revolution of the area between the curve $y = f(x)$, the x-axis and two ordinates $x = a$ and $x = b$ is given by $V = \left| \int_a^b \pi y^2 \, dx \right|$
- (b) The volume of the solid generated by the revolution of the area between the curve $x = f(y)$, the y-axis and two abscissa $y = a$ and $y = b$ is given by $V = \left| \int_a^b \pi x^2 \, dy \right|$
- (c) The volume of the solid generated by the revolution of the area bounded by two curves $y_1 = f(x)$ and $y_2 = g(x)$ and two abscissa $x = a$ and $x = b$ about x-axis is $\left| \int_a^b \pi (y_1^2 - y_2^2) \, dx \right|$
- (d) The volume of the solid generated by the revolution of the area bounded by two curves $x_1 = f(y)$ and $x_2 = g(y)$ and two ordinates $y = a$ and $y = b$ about y-axis is $\left| \int_a^b \pi (x_1^2 - x_2^2) \, dy \right|$
- (e) **EVEN FUNCTION:** A function $f(x)$ is said to be even function if $f(-x) = f(x)$, for all x
For example: $x^2, x^4, x^2 + x^6, \cos x$
- (f) **ODD FUNCTION:** A function $f(x)$ is said to be odd function if $f(-x) = -f(x)$, for all x
For example: $x, x^3, x + x^3, \sin x$



SOME USEFUL TRIGONOMETRIC IDENTITIES

1) $\sin^2 x + \cos^2 x = 1$

2) $\sec^2 x - \tan^2 x = 1$

3) $\operatorname{cosec}^2 x - \cot^2 x = 1$

4) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

5) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

6) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

7) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

8) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

9) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

10) $\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

11) $\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

12) $\sin 2A = 2 \sin A \cos A$

13) $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$

14) $\cos 2A = \cos^2 A - \sin^2 A$

15) $\cos 2A = 1 - 2 \sin^2 A$ (or) $\sin^2 A = \frac{1 - \cos 2A}{2}$

16) $\cos 2A = 2 \cos^2 A - 1$ (or) $\cos^2 A = \frac{1 + \cos 2A}{2}$

17) $\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$

18) $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

19) $\sin 3A = 3 \sin A - 4 \sin^3 A$ (or) $\sin^3 A = \frac{3 \sin A - \sin 3A}{4}$

20) $\cos 3A = 4 \cos^3 A - 3 \cos A$ (or) $\cos^3 A = \frac{\cos 3A + 3 \cos A}{4}$

21) $\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

22) $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

23) $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$

24) $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

25) $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$



UNIT-V

1. Mean and RMS values

- 1.1 **Mean value:** The mean value (MV) of the function $y=f(x)$ over the interval $[a, b]$ is defined as

$$\begin{aligned} MV &= \frac{1}{b-a} \int_a^b y \, dx \\ &= \frac{1}{b-a} \int_a^b f(x) \, dx \end{aligned}$$

- 1.2 **Mean square value of function:** The Mean Square value (MSV) of the function $y=f(x)$ over the interval $[a, b]$ is defined as

$$\begin{aligned} MSV &= \frac{1}{b-a} \int_a^b y^2 \, dx \\ &= \frac{1}{b-a} \int_a^b [f(x)]^2 \, dx \end{aligned}$$

- 1.3 **Root Mean Square (RMS) value:** The Root Mean Square (RMS) value of the function $y=f(x)$ over the interval (a, b) is defined as

$$\begin{aligned} RMS \, Value &= \sqrt{\frac{1}{b-a} \int_a^b y^2 \, dx} \\ &= \sqrt{\frac{1}{b-a} \int_a^b [f(x)]^2 \, dx} \end{aligned}$$

2. Numerical Integration

2.1 Trapezoidal Rule:

$$\int_a^b y \, dx \approx \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots + y_{n-1})]$$

where n is any positive integer, $h = \frac{b-a}{n}$ and $y_0, y_1, y_2, \dots, y_n$ are the values of y when $x = x_0, x_1, x_2, \dots, x_n$ (where $x_0 = a, x_n = b$)



2.2 Simpson's Rule:

$$\int_a^b y \, dx \approx \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \dots + y_{n-2})]$$

where n is any positive **EVEN** integer, $h = \frac{b-a}{n}$ and $y_0, y_1, y_2, \dots, y_n$ are the values of y when $x = x_0, x_1, x_2, \dots, x_n$ (where $x_0 = a, x_n = b$)

UNIT-VI

Differential Equations of first order

1. **Formation of differential Equation-**The differential equation can be formed by eliminating arbitrary constants using differentiation.

2. **Solution of differential Equation by**

- (a) Variables separable
- (b) Homogeneous Equation
- (c) Exact differential Equation
- (d) Linear differential equations
- (e) Bernoulli's equation

2.1 **Solution of DE by variables separable:**

If a differential equation can be expressed in the form $f(x) \, dx + g(y) \, dy = 0$ or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ then such a form is called variables separable.

The General Solution of $f(x) \, dx + g(y) \, dy = 0$ is $\int f(x) \, dx + \int g(y) \, dy = c$

Where 'c' is an arbitrary constant.

2.2 **Homogeneous differential equation**

A function $f(x, y)$ is said to be a homogeneous function in x and y of degree n, if $f(kx, ky) = k^n f(x, y)$, for all values of k where n is constant.

- (a) Put the given equation in the form of $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ or $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ where $f(x, y)$ and $g(x, y)$ are both Homogeneous functions of same degree.
- (b) Put $y = vx$ so that $\frac{dy}{dx} = v + x \frac{dv}{dx}$
- (c) Solve the new differential equation by variable separable method and substitute $v = \frac{y}{x}$



2.3 Exact differential equation

The DE $M(x,y)dx + N(x,y)dy = 0$ is said to be exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Its general solution is

$$\int_{(y \text{ constant})} M dx + \int_{(no \ x \ terms)} N dy = C$$

(OR)

- $\int M(x,y)dx$ by treating y as a constant
- $\int N(y)dy$ take the terms free from x
- The solution of exact differential equation is $\int M(x,y)dx + \int N(y)dy = c$

2.4 Linear differential equation (LDE)

The differential equation is of the form $\frac{dy}{dx} + Py = Q$ is called LDE, Where P and Q are either constants or functions of x .

Integrating Factor (I.F) $= e^{\int P dx}$

General Solution (GS) of LDE is

$$y (I.F) = \int Q (I.F) dx + c$$

2.5 Bernoulli's equation (Reducible to LDE)

The differential equation of the form $\frac{dy}{dx} + Py = Qy^n$ is called Bernoulli's differential equation where P and Q are either constants or functions of x and $n \neq 0, n \neq 1$

- Divide the given equation by y^n
- Put $y^{1-n} = v$ and get the Linear equation in v
- Solve the Linear equation in v
- Substitute v to obtain general solution.